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The Entropy Density of Quasi Free States

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Abstract. For lattice invariant quasi free states on the Fermi lattice system the mean entropy is explicitly calculated; it is proved that it is a norm continuous functional on this set of states which is not weakly continuous.

I. Introduction

In the algebraic approach of statistical mechanics a lot of work has been done on the existence and properties of the mean entropy of translation invariant states (Ref. [1, 2]). For classical systems and quantum lattice systems the existence of mean entropy has been proved; furthermore as fundamental properties it has been proved that the entropy-functional on the set of invariant states is affine and semi-continuous.

In this work we study the mean entropy of the quasi-free lattice invariant states on the Fermi lattice system. The existence of the mean entropy follows from Ref. [2]. Here we derive an explicit form of the mean entropy for those states (Theorem 2). This result may be usefull in deriving rigorous results for solvable models in statistical mechanics.

Furthermore much attention is given to the continuity properties of the entropy functional with respect to different topologies on the set of quasi-free states. It is proved that it is continuous with respect to the norm topology (Theorem 1) and that it is not continuous with respect to the weak topology (Theorem 3).

II. Entropy density

A. Preliminaries

1. The Fermi Lattice Algebra

We consider the one-dimensional lattice $\mathbb{Z} = \{0, \pm 1, \pm 2, \ldots\}$. The Hilbert space $l^2(\mathbb{Z})$ can also be considered as an Euclidian space H

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