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There are no Goldstone Bosons in Two Dimensions*

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Abstract. In four dimensions, it is possible for a scalar field to have a vacuum expectation value that would be forbidden if the vacuum were invariant under some continuous transformation group, even though this group is a symmetry group in the sense that the associated local currents are conserved. This is the Goldstone phenomenon, and Goldstone's theorem states that this phenomenon is always accompanied by the appearance of massless scalar bosons. The purpose of this note is to show that in two dimensions the Goldstone phenomenon can not occur; Goldstone's theorem does not end with two alternatives (either manifest symmetry or Goldstone bosons) but with only one (manifest symmetry).

Introduction and Conclusions

One of the oldest versions of Goldstone's theorem [1] is this: Let ϕ be a local scalar field, and let j_μ be a local conserved vector [2] current,

$$\partial^\mu j_\mu = 0 . \tag{1}$$

Define the scalar field $\delta\phi$ by

$$\delta\phi(y) = i \int d^3 \boldsymbol{x} [j_0(x_0, \boldsymbol{x}), \phi(y)] . \tag{2}$$

(Because of locality – the statement that the fields and currents commute at space-like separations – the integrand always vanishes for sufficiently large x; thus the integral always converges. Also, as a consequence of Eq. (1), this object is independent of x_0 and defines a scalar field [3].) Then, assuming the usual axioms of field theory [4], either (1) the vacuum expectation value of $\delta\phi$ vanishes, or (2) the theory contains a massless scalar meson (the Goldstone boson).

(Formally, $\delta\phi$ is the commutator of ϕ with the global conserved charge associated with the local conserved current, and the non-vanishing of its vacuum expectation value implies that the charge does not annihilate the vacuum, or (formally equivalently) that the vacuum

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