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Tilted Homogeneous Cosmological Models

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Abstract. We examine spatially homogeneous cosmological models in which the matter content of space-time is a perfect fluid, and in which the fluid flow vector is not normal to the surfaces of homogeneity. In such universes, the matter may move with non-zero expansion, rotation and shear; we examine the relation between these kinematic quantities and the Bianchi classification of the symmetry group. Detailed characterizations of some of the simplest such universe models are given.

1. Introduction: Covariant Formalism

In a previous series of papers ([1–3]), exact solutions of Einstein's field equations

$$R_{ab} - \tfrac{1}{2} R g_{ab} + \Lambda g_{ab} = T_{ab} \tag{1.1}$$

were studied under the assumptions that

(1) the matter takes the "*perfect fluid*" form:

$$T_{ab} = \mu u_a u_b + p(g_{ab} + u_a u_b), \; u_a u^a = -1, \; \mu > 0, \; p \geqq 0 \tag{1.2}$$

where u^a is the fluid 4-velocity, μ the energy density and p the pressure;

(2) space-time is locally invariant under a group of isometries G_3 simply transitive on spacelike surfaces $S(t)$, i.e. space-time is *spatially homogeneous*;

(3) the 4-velocity u^a is everywhere orthogonal to the homogeneous surfaces $S(t)$.

In this paper, we study a wider class of spatially homogeneous cosmological models: we maintain conditions (1) and (2), but drop (3). This allows a wider variety of behaviour, for when (3) is dropped, the fluid may have non-zero vorticity and acceleration. Further, such a universe may appear to be inhomogeneous to a fundamental observer (e.g. number counts of radio sources or galaxies will look inhomogeneous) even though the space-time and its contents are spatially homogeneous in a strict mathematical sense. Our discussion supplements previous discussions of such universes (see e.g. [4–6]).

The immediate geometrical objects defined in a space-time in which (1) and (2) hold are the surfaces of homogeneity $S(t)$ and the fluid flow