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# Decrease Properties of Truncated Correlation Functions and Analyticity Properties for Classical Lattices and Continuous Systems

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**Abstract.** We present and discuss some physical hypotheses on the decrease of truncated correlation functions and we show that they imply the analyticity of the thermodynamic limits of the pressure and of all correlation functions with respect to the reciprocal temperature $\beta$ and the magnetic field $h$ (or the chemical potential $\mu$) at all (real) points $(\beta_0, h_0)$ (or $(\beta_0, \mu_0)$) where they are supposed to hold. A decrease close to our hypotheses is derived in certain particular situations at the end.

## Introduction

In a recent work [1], Lebowitz considers a ferromagnetic Ising spin lattice and presents a method which, starting from decrease properties of the correlations, proved in certain cases, allows him to derive regularity properties of the infinite-volume free energy $\psi(\beta, h)$ and correlations $\langle \sigma_A \rangle (\beta, h)$, at real points $(\beta_0, h_0)$ where other methods have given so far no information (e.g. at points close to $\beta_c$). However, even for finite range potentials, only infinite differentiability, rather than analyticity has been obtained. The purpose of the present paper is to present and discuss certain "physically reasonable" hypotheses, not proved in general so far, on the decrease of the truncated functions[1] and to show that they do yield analyticity.

[1] Also called "cluster functions" or "Ursell functions" of the correlations in statistical mechanics.