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Return to Equilibrium

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Abstract. The problem of return to equilibrium is phrased in terms of a C^*-algebra $\mathfrak{A}$, and two one-parameter groups of automorphisms τ, τ^P corresponding to the unperturbed and locally perturbed evolutions. The asymptotic evolution, under τ, of τ^P-invariant, and τ^P-K.M.S., states is considered. This study is a generalization of scattering theory and results concerning the existence of limit states are obtained by techniques similar to those used to prove the existence, and intertwining properties, of wave-operators. Conditions of asymptotic abelianness provide the necessary dispersive properties for the return to equilibrium. It is demonstrated that the τ^P-equilibrium states and their limit states are coupled by automorphisms with a quasi-local property; they are not necessarily normal with respect to one another. An application to the $X - Y$ model is given which extends previously known results and other applications, and examples, are given for the Fermi gas.

I. Introduction

We examine general properties of systems whose dynamics have been locally perturbed and illustrate these properties with examples. Our specific interest is whether systems, that have been perturbed in this manner, return to equilibrium under the unperturbed evolution. In this context we consider the behaviour of states which are invariant, or satisfy the K.M.S. condition, for the perturbed dynamics. We demonstrate that this type of problem is tractable with methods which are a natural generalization of scattering theory.

For simplicity of formulation we work in an algebraic setting and assume that the kinematic observables of our system form a C^*-algebra $\mathfrak{A}$. The dynamics is specified by a one-parameter group of automorphisms τ of $\mathfrak{A}$ which we take to be strongly continuous, i.e.

$$\|\tau_t(A) - A\| \xrightarrow[|t|=0]{} 0 .$$

These assumptions could be weakened at the cost of introducing more detailed structure which is not directly relevant to the problem under discussion.

In Section 2 we define a second group of automorphisms τ^P of $\mathfrak{A}$ which is to be considered as a group arising from a local perturbation of the Hamiltonian. We point out some properties which are stable