The Four Laws of Black Hole Mechanics

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Abstract. Expressions are derived for the mass of a stationary axisymmetric solution of the Einstein equations containing a black hole surrounded by matter and for the difference in mass between two neighboring such solutions. Two of the quantities which appear in these expressions, namely the area A of the event horizon and the "surface gravity" κ of the black hole, have a close analogy with entropy and temperature respectively. This analogy suggests the formulation of four laws of black hole mechanics which correspond to and in some ways transcend the four laws of thermodynamics.

1. Introduction

It is generally believed that a gravitationally collapsing body will give rise to a black hole and that this black hole will settle down to a stationary state. If the black hole is rotating, the stationary state must be axisymmetric [1] (An improved version of this theorem involving weaker assumptions is outlined in [2] and is given in detail in [3]). It has been shown that stationary axisymmetric black hole solutions which are empty outside the event horizon fall into discrete families each of which depends on only two parameters, the mass M and the angular momentum J [4–6]. The Kerr solutions for $M^4 > J^2$ are one such family. It seems unlikely that there are any others. It also seems reasonable to suppose that the Newman-Kerr solutions for $M^4 > J^2 + M^2 Q^2$, where Q is the electric charge, are the only stationary axisymmetric black hole solutions which are empty outside the event horizon apart from an electromagnetic field. On the other hand there will be an infinite dimensional family of stationary axisymmetric solutions in which there are rings of matter orbiting the black hole. In Sections 2 and 3 of this paper we shall derive formulae for the mass of such a solution and for the difference in mass of two nearby solutions. These formulae

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