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Remarks Concerning the Connection between Properties of the 4-Point-Function and the Wilson-Zimmermann Expansion

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Abstract. This paper concerns an investigation of the Wilson-Zimmermann (or "short distance") expansion for $A(x)\,A(y)$ with $x\to y$ where $A(x)$ is a real scalar field fulfilling Wightman's axioms. If one assumes that such an expansion exists, where the terms of the expansion are operators relatively local to $A(x)$, then the singularities arising in the 4-point-function for $x_3\to x_4$ must control the singularities of the n-point functions ($n=4,5,6,\ldots$) arising for $x_j\to x_{j+1}$, $j=1,2,\ldots,n-1$. A similar consequence can be drawn if the terms of the expansion are assumed to exist only as bilinear-forms (Section 2). For certain classes of fields one can show that this condition necessary for the short distance expansion is indeed fulfilled (Section 3). The result of the last section is that the above mentioned condition is also sufficient for the Wilson-Zimmermann expansion, interpreted as an expansion into bilinear forms, and also as an operator expansion in a somewhat modified sense.

1. Introduction

A field theory in which the principle of contact interaction shall be valid needs for the definition of the interaction terms products of field quantities taken at the same position.

If one intends to apply this principle also in a relativistic quantum field theory one has to define products of field operators at the same position. The difficulties which arise in this procedure are very well known. They have their origin in the distributive character of the field operators.

In the case of the free field $A_0(x)$ with $x=(x_0,x_1,x_2,x_3)$ however it is well known how one can get $A_0^n(x)$, $n=2,3,4\ldots$ in the form of Wick products of the field, by subtracting products of 2-point-functions W_2^0. The $A_0^n(x)$ are again tempered distributions which are relatively local to $A_0(x)$, this means

$$[A_0^n(x),A_0(y)]=0 \quad \text{for } x-y \text{ spacelike}\,.$$

In the last years a heuristic ansatz for the products of field-operators at the same position in the case for interacting fields has been proposed