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Correlation Inequalities and the Mass Gap in $P(\varphi)_2$

I. Domination by the Two Point Function

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Abstract. For the $P(\varphi)_2$ field theory, we prove that the falloff of the (vacuum subtracted) two point Schwinger function dominates the higher order (vacuum subtracted) Schwinger functions. As applications, we prove that for even polynomials, the first excited state is odd, and that when there is a one particle state in the infinite volume limit, it is coupled to the vacuum by a single power of the field. The main inputs are the theory of Markov fields and the F.K.G. inequalities.

1. Introduction

In this note, we discuss certain properties of the $P(\varphi)_2$ theory of Glimm, Jaffe, and Rosen (see [2, 3, 20] for references and background). We will employ the statistical mechanical techniques [22, 4] made available by Nelson's Markov field theory [12, 13, 15]. In fact, our line of approach is suggested by some recent work of Lebowitz [9] on the properties of ferromagnetic Ising models. Lebowitz proved that the rate of decrease of the two point spin correlations, $\langle\sigma_i\sigma_j\rangle - \langle\sigma_i\rangle\langle\sigma_j\rangle$, as $|i-j|\to\infty$ is no faster than that of any other spin-spin correlation $\langle\sigma_{i_1}\dots\sigma_{i_m}\sigma_{j_1}\dots\sigma_{j_k}\rangle - \langle\sigma_{i_1}\dots\sigma_{i_m}\rangle\langle\sigma_{j_1}\dots\sigma_{j_k}\rangle$ as $\min|i_p - j_q|\to\infty$. In the language of transfer matrices [16, 10, 11], this says that a single spin must have a non-vanishing matrix element between the two lowest eigenvectors. Our goal here will be to prove analogous results for the $P(\varphi)_2$ field theory. As explained in [5], the (spatially cutoff or infinite volume) Hamiltonian plays the role of a transfer matrix in Markov field theory. In fact, this analogy is the basis of (and is implicit in) the proof of Nelson's reconstruction theorem [12].

Our main theorems appear in Sections 5–7. They are essentially corollaries of a technical result in Section 4 which is very similar to Lebowitz'main technical estimate (Lemma 1 of [9]). Lebowitz relied on the correlation inequalities proven for ferromagnetics by Fortuin,

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