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Existence of Green's Functions for Dilute Bose Gases

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Abstract. Some properties of finite volume Green's functions are obtained, and the infinite volume limit is shown to exist for the multi-time Green's functions of a dilute Bose gas, constructed with the operators of the quasi-local algebra (see Theorem IV.5).

0. Introduction and Notations

For our study of a quantum system of identical bosons, we need:

— A Fock-space $\mathscr{H}_\Lambda$, associated to each bounded open set Λ of the configuration space $\mathbb{R}^\nu : \mathscr{H}_\Lambda = \bigoplus_{n \geqq 0} \mathscr{H}_\Lambda^{(n)}$, where $\mathscr{H}_\Lambda^{(n)} = L_+^2(\Lambda^n)$ is the Hilbert space of complex square integrable functions with support in Λ^n, symmetric with respect to the arguments.

— A Hamiltonian H_Λ, defined from a two-body potential ϕ depending on the relative position $x \in \mathbb{R}^\nu$, and satisfying the following properties:

1) ϕ is a real function, continuous outside of the origin (everywhere if $\nu = 1$).

2) ϕ is stable: $\exists\, B \geqq 0$ such that $\forall\, n > 0$, $\forall\, x_1, \ldots, x_n \in \mathbb{R}^\nu$:

$$U(x_1, \ldots, x_n) = \sum_{i<j} \phi(x_i - x_j) \geqq -n \cdot B .$$

3) In the end of part III, and in part IV for Theorems 2 and following we need also that ϕ is square integrable in the whole space. Then for each $n \geqq 0$, $H_\Lambda^{(n)}$, formally equal to $-\mu \cdot n + \sum_{i=1}^{n} \frac{p_i^2}{2m} + U(x_1, \ldots, x_n)$ is a self-adjoint operator with domain $\mathscr{D}(H_\Lambda^{(n)})$, bounded from below, defined by use of the Wiener integral and the Feynmann-Kac formula [1]. Now an essentially self-adjoint operator is defined by $\sum_{n=0}^{N} \psi^{(n)} \to \sum_{n=0}^{N} H_\Lambda^{(n)} \psi^{(n)}$ where $N < \infty$ and $\psi^{(n)} \in \mathscr{D}(H_\Lambda^{(n)})$, and the Hamiltonian H_Λ is the closure of this operator with domain $\mathscr{D}(H_\Lambda) = \left\{ \psi \in \mathscr{H}_\Lambda : \sum_{n \geqq 0} \|H_\Lambda^{(n)} \psi^{(n)}\|^2 < \infty \right\}$.