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Elastic Perturbation Theory in General Relativity and a Variation Principle for a Rotating Solid Star

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Abstract. Perturbation analysis is applied to the theory of a General Relativistic perfectly elastic medium as developed by Carter and Quintana (1972). Formulae are derived for the Eulerian variations of the principal fields (density, pressure tensor, etc.) on which the description of such a medium is based, where the perturbations are induced both by infinitesimal displacements of the medium and by infinitesimal variations of the metric tensor. These formulae will be essential for problems such as the study of torsional vibration modes in a neutron star.

As examples of their application, the variation formulae are used in the derivation firstly of a simple (dynamic) action principle for a perfectly elastic medium (this principle being a generalisation of the one given by Taub (1954) for a perfect fluid) and secondly in the derivation of a rather more sophisticated mass variation principle for a stationary rotating solid star (this principle being a generalisation of the one given by Hartle and Sharp (1967) for a perfect fluid star).

1. Introduction

In the general theory of perfectly elastic solids (as compared with the special subcase of a perfectly elastic fluid) the *linearised* theory of small deformations plays a disproportionately important role, since in most solid materials the behaviour will be on the point of ceasing to be perfectly elastic (due to fracture or hysteresis effects) when the deformations are sufficiently large for deviations from linearity to be important.

The primary purpose of the present article is to show how to calculate the linearised perturbations in the fundamental tensor fields used to describe a perfectly elastic medium in the General Relativistic theory recently developed by Carter and Quintana (1972) [1]. In Newtonian elasticity theory it is usual to consider all elastic perturbations as being due to displacements with respect to the (flat) background space. In general relativity theory it is necessary also to take into account the effect of geometric changes due to absolute variations of the space-time metric tensor. Indeed in a fully covariant theory it is possible in principle to consider *all* variations as being due to changes in the geometry of