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Propagation of Shock Waves in Interacting Higher Spin Wave Equations

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Abstract. The derivation of characteristic surfaces for interacting higher spin wave equations is discussed in a shock wave formalism. Equations describing the propagation of shock waves along the bicharacteristics are established for several interacting systems.

1. Introduction

The programme of constructing Lagrangians to give a consistant field theoretical description of higher spin particles ($s \geqq 1$) was initiated by Fierz and Pauli [1]. The resulting Lagrange equations are then equivalent to the equations of motion of the field together with the constraints necessary to reduce the number of degrees of freedom of the field to the number specified by its spin. This method eliminates the algebraic inconsistencies which may arise when an interaction is introduced and the constraint equations are postulated independently of the equations of motion.

But it was noticed by Johnson and Sudarshan [2] that when the Rarita-Schwinger field for a spin-3/2 particle is coupled minimally to the electromagnetic field, a peculiar anomaly appears: namely, despite the addition of a relativistically invariant interaction term to the free Lagrangian, the equal time commutation relation between fields is not positive definite in all Lorentz frames. Subsequently, Velo and Zwanziger [3–5] showed that anomalies appear in the first quantized versions of the same, and other, interacting systems by demonstrating, for example [3], that the minimally coupled Rarita-Schwinger equation has characteristic surfaces which can be space-like for any non-vanishing value of the Maxwell tensor. Their method involves considering the Cauchy problem and using the definition of characteristic surfaces as those initial surfaces for which it has not a unique solution [6].

In this paper we use the fact that characteristic surfaces are surfaces across which there can exist discontinuities in the highest order derivatives appearing in the wave equation. This method was first used by Stell-

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