Commun. math. Phys. 30, 171—200 (1973)

Uniqueness of the Physical Vacuum and the Wightman Functions in the Infinite Volume Limit for Some non Polynomial Interactions

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Received August 31; in revised form December 5, 1972

Abstract. We consider quantum field theoretical models in n dimensional space-time given by interaction densities which are bounded functions of an ultraviolet cut-off boson field. Using methods of euclidean Markov field theory and of classical statistical mechanics, we construct the infinite volume imaginary and real time Wightman functions as limits of the corresponding quantities for the space cut-off models. In the physical Hilbert space, the space-time translations are represented by strongly continuous unitary groups and the generator of time translations H is positive and has a unique, simple lowest eigenvalue zero, with eigenvector Ω, which is the unique state invariant under space-time translations. The imaginary time Wightman functions and the infinite volume vacuum energy density are given as analytic functions of the coupling constant. The Wightman functions have cluster properties also with respect to space translations.

1. Introduction

In recent years the mathematical construction of quantum field theoretical models has made an impressive progress [1]. For the polynomial interactions [2] in two-dimensional space-time all the Haag-Kastler axioms for a quantum field theory of local observables have been verified, as well as most of the Wightman axioms [3].

In particular in these polynomial models (and also for certain 2-dimensional boson models with exponential interactions [4]) the existence of a vacuum state has been proven [4].

This was sufficient for Glimm and Jaffe to build a theory in which the Wightman functions exist and have some of the important physical properties embodied in Wightman's axioms.

The question of the uniqueness of the vacuum has not been tackled yet. The vacuum state is only obtained by a compactness argument as

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[1] See e.g. [1] and the references given therein.

[2] See e.g. [1, 2] and the references given therein.

[3] See e.g. [1–3]. See also footnote 5 below.

[4] This has been proven also for the two-dimensional Yukawa interaction [5].