Commun. math. Phys. 30, 99—128 (1973)

A Finite-dimensional Canonical Formalism in the Classical Field Theory

Jerzy Kijowski
Institute of Mathematical Methods of Physics, Warsaw University, Warsaw, Poland

Received March 8; in revised form November 1, 1972

Abstract. A canonical formalism based on the geometrical approach to the calculus of variations is given. The notion of multi-phase space is introduced which enables to define whole the canonical structure (physical quantities, Poisson bracket, canonical fields) without use of functional derivatives. All definitions are of pure geometrical (finite dimensional) character.

The observable algebra $\mathcal{O}$ (physical quantities algebra) obtained here is much smaller then the algebra of all (sufficiently smooth) functionals on the space of states, derived from the standard infinite-dimensional formulation. As it is known, the latter is much too large for purposes of quantization. As the examples prove, our algebra $\mathcal{O}$ could be an adequate start-point for quantization.

For simplifying the language the notion of observable-valued distribution is introduced. Many concrete physical examples are given. E.g. it is shown that some problems connected with gauge in electrodynamics are automatically solved in this approach. The introduced language allows to obtain the Noether theorem in a most natural way.

1. Introduction

The present state of quantization of non-linear fields theories (cf. [1]) may lead to the conclusion that there may be more deep differences between linear and non-linear theories that one may infer from the usual canonical formalism, based on the following analogy with classical mechanics:

Mechanics	Field theory
Time	Time
Finite dimensional space of all possible positions at given time	Infinite dimensional space of states of a field at given time

This formalism, initially using notions not too precise from the mathematical viewpoint (e.g. multiplication of functional derivatives) has now acquired a fine mathematical formulation (theory of infinite dimensional symplectic spaces – cf. [11] and [12]). It leads to very rich observable algebra (algebra of physical quantities), Poisson bracket, canonical fields etc. Linear theories are not distinguished in this formalism.