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Correlation Functions and the Uniqueness of the State in Classical Statistical Mechanics*

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Abstract. A general criterion is derived which assures the uniqueness of the state of a classical statistical mechanical system in terms of a given system of correlation functions. The criterion is $\sum_k (m_{k+j}^A)^{-1/k} = \infty$ for all j and all bounded sets A, where

$$m_k^A = (k!)^{-1} \int_A \cdots \int_A \varrho_k(x_1, \ldots, x_k)\, dx_1 \ldots dx_k .$$

1. Introduction

In the older literature of classical statistical mechanics it was taken for granted, although not explicitly stated, that the sequence of *correlation functions* $\varrho_n, n = 1, 2, 3, \ldots$, in their totality uniquely characterize the (statistical) state of the system to which they refer. That this is not the case in general was pointed out by Ruelle[1]. Thus, the problem arises of supplying criteria under which the uniqueness of the state is guaranteed. Such a criterion was already given in Ruelle's book, namely the existence of a positive constant c such that $|\varrho_n(x_1, x_2, \ldots, x_n)| \leqq c^n$ for all n and almost all values of the variables. Nevertheless, the question is interesting enough to merit more detailed investigation; the present paper is devoted to this task.

Before one can attack the problem it is necessary to specify precisely the mathematical set-up in whose context the question is posed. In a quite general manner, we do this as follows. We consider a space X (the analogue of Gibbs's phase space) whose points ξ are infinite "particle configurations" in a space E (the "one particle space"). A natural measure theoretical structure is defined in X. A *state* of the system is taken to mean a probability measure μ over X. Since the correlation functions ϱ_n

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[1] Ref. [4], p. 103 and Exercise 4E.