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On the Uniqueness of the Equilibrium State for Antiferromagnetic Ising Spin System in the Phase Transition Region

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Abstract. We show that at low temperature an Ising spin system with antiferromagnetic interaction in a small enough external magnetic field has only one translationally invariant state.

Introduction

We consider an Ising antiferromagnet with nearest neighbour interaction in a finite box Λ on a two-dimensional lattice $\mathbb{Z}^2$ i.e. at each point x_i of the lattice there is a spin $\sigma_{x_i} = \pm 1$. The conditional probability of a spin-configuration $\{\underline{\sigma}\}$ in Λ for a given boundary configuration $\underline{\tau}$ is proportional $e^{-\beta H_\Lambda(\underline{\sigma})}$ where

$$H_\Lambda(\underline{\sigma}) = J \sum_{\substack{\langle i,j\rangle \\ i \neq j}} (\sigma_i \sigma_j + 1) - h \sum_i \sigma_i + J \sum_{\substack{\langle i,j\rangle \\ i \in \Lambda \\ j \notin \Lambda}} (\sigma_i \tau_j + 1) \tag{0.1}$$

τ_j belongs to the first external layer, J is pair interaction, h is an external magnetic field, β is the reciprocal temperature. A boundary condition for the system in the box Λ is specified by giving a probability distribution $P_\Lambda(\underline{\tau})$ for the boundary configuration $\underline{\tau}$.

An (equilibrium) state of the infinite system is defined to be a family of correlation functions $\langle \sigma_S \rangle_{h,\beta}$, for the finite subset S of $\mathbb{Z}^2$, obtained as suitable thermodynamical limit of

$$\langle \sigma_S \rangle_{\Lambda,h,\beta,P_\Lambda} \equiv \left\langle \prod_{x_i \in S} \sigma_{x_i} \right\rangle_{\Lambda,h,\beta,P_\Lambda} \quad \text{with} \quad S \subseteq \Lambda \quad \text{and}$$

$$\langle \sigma_S \rangle_{\Lambda,h,\beta,P_\Lambda} = \sum_{\underline{\tau}} P_\Lambda(\underline{\tau}) \langle \sigma_S \rangle_{\Lambda,\beta,h,\underline{\tau}}$$

i.e. of spin-correlation functions for a sequence of finite boxes with some boundary condition P_Λ.

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