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Van der Waals-Maxwell Theory, Lebowitz-Penrose Limit and Superstable Interactions

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Abstract. We obtain rigorous results about the liquid-vapour phase transition. We prove that in the Lebowitz-Penrose limit the hypothesis of hard core for the interaction is not essential to reproduce the Van der Waals-Maxwell theory. It can be replaced by the superstability.

1. Introduction

Many efforts have been devoted to the deduction of the features of liquid-vapor phase transition from the fundamental principles of statistical mechanics. Some authors have treated the problem by introducing a long range interaction [1–4]. In particular Lebowitz-Penrose (henceforth referred to as LP) have considered a system of classical particles interacting pairwise in ν dimensions via a potential of the form

$$v(r,\gamma)=q(r)+\gamma^{\nu}\varphi(\gamma r) \tag{1.1}$$

where r is the vector distance between a pair of particles, and γ is a positive parameter; $q(r)$ is the "reference potential": it has a hard core and decreases in an integrable way to infinity; $\gamma^{\nu}\varphi(\gamma r)$ is the "Kac potential": γ fixes simoultaneously the range and the strength.

LP have studied the thermodynamic functions in the limit $\gamma\to 0$. Namely they first carried out the thermodynamic limit and then considered the range of the Kac potential increasing to infinity. The most interesting Kac potentials from the physical point of view are the non-repulsive one, i.e. $\varphi\leqq 0$; in this case LP have shown that

$$a(\varrho,0^{+})=\lim_{\gamma\to 0}a(\varrho,\gamma)=\mathrm{CE}\left(a^{0}(\varrho)+\tfrac{1}{2}\alpha\varrho^{2}\right) \tag{1.2}$$

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