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Trilinear Lorentz Invariant Forms

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Abstract. Trilinear invariant forms are described over spaces transforming under the so-called elementary representations of $SL(2, C)$ obtained from the Gel'fand-Naimark principal series by analytic continuation in the representation parameters (among these are all infinite-dimensional completely irreducible representations). All such forms are described using a manifestly covariant technique. The method is based on a natural one-one correspondence between the invariant forms and invariant separately homogeneous distributions (called kernels of the forms) in three complex two-dimensional non-zero vectors; thus the problem is completely reduced to a problem of distribution theory. The kernels display analyticity properties in the representation parameters; the results on this point are only sketched.

0. Introduction

0.1. Distribution Theoretic Formulation of the Problem

The problem on continuous polylinear invariant forms over spaces transforming under elementary representations[1] of the connected Lorentz group $\mathscr{L}_+^\uparrow$ (or of its universal covering group, $SL(2, C)$, consisted of all complex unimodular 2×2 matrices) has been raised in [2]. The interest in the forms arises from the fact that these provide powerful tools in studying elementary representations. For example, through the use of the bilinear invariant forms (which have been studied thoroughly in [2]) one can determine intertwining operators, equivalence conditions, existence of invariant pre-Hilbert structures, etc. The present paper is devoted to description of continuous trilinear invariant forms (over the spaces mentioned above). The importance of the trilinear forms is in their intimate connection with analysis of tensor product of two elementary (and, in particular, of infinite-dimensional completely irreducible[2]) representations of the Lorentz group (see [3] and Remark

[1] For the definition and general properties of elementary representations of a complex semi-simple Lie group, we refer to [1]. The case of $SL(2, C)$ which is our main concern is treated in great detail in [2].

[2] A representation of a group in a topological vector space is said to be completely irreducible if the weakly closed linear hull of the representation operators contains all continuous operators in the representation space [1]. The complete irreducibility implies the topological and the operator irreducibility.