Commun. math. Phys. 29, 177—187 (1973)

On Renormalization and Complex Space-Time Dimensions

J. F. Ashmore

International Atomic Energy Agency
and
United Nations Educational Scientific and Cultural Organization
International Centre for Theoretical Physics, Trieste, Italy

Received July 31, 1972

Abstract. The method of using the dimension of space-time as a complex parameter introduced recently to regularize Feynman amplitudes is extended to an arbitrary Feynman graph. The method has promise of being particularly well-suited to gauge theories. It is shown how the renormalized amplitude, together with the Lagrangian counter-terms, may be extracted directly, following the method of analytic renormalization.

I. Introduction

Of late, a number of authors suggested, independently, an approach to the renormalization of the perturbation expansion in Lagrangian quantum field theory which uses the dimension of space-time as a complex parameter [1–3]. The most important feature of the method is that the regularization procedure in general preserves the formal structure necessary for the theory to satisfy Ward-Takahashi identities appropriate to the gauge symmetries present. With exceptions, the argument relies on the observation that the Ward-Takahashi identities are *formally* independent of the space-time dimension[1]. The method of extracting renormalized results is very close in spirit to the method of analytic renormalization of Speer [4, 5], which, *per se*, does not preserve gauge symmetries [6].

Now the attractive feature of analytic renormalization is that the renormalized amplitude is defined *non-recursively*, but it is equivalent to the additive, *recursive* definition of Bogolubov, Parasiuk, and Hepp [7] (which is the most general treatment in Lagrangian quantum field theory). Both approaches have been shown to yield the Lagrangian counter-terms directly.

In Refs. [1–3], the method of regularization was only demonstrated by example in lowest orders. It is our purpose here to give a general

[1] The exceptions are, for example, when the theory contains an axial coupling; it is then implicitly necessary that there be an odd number of space dimensions.