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Dynamical Theory of a Bidimensional System with an Infinite Number of Degrees of Freedom

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Abstract. The dynamical theory of a bidimensional model of hard squares with elastic collisions is presented. The time evolution is shown to exist on a large class of infinite configurations. Moreover, it is proved that any equilibrium state, that is any solution of the equilibrium equations, is concentrated on this set of allowed initial configurations and is invariant under the time evolution.

§ 1. Introduction

This paper is devoted to proving some results concerning the dynamical theory of a bidimensional system with infinitely many degrees of freedom. Namely we consider infinite configurations of particles that can be regarded as hard squares of a common size. As indicated in Fig. 1, these squares are restricted to having their sides parallel to the x- and y-axis. Their velocities are all equal to v_0 in absolute values and parallel to any of the two bissectrix of (Ox, Oy), so that there are only four possible velocities for each particle. The elastic collisions between the squares will be the only interactions considered.

Let us make some remarks to explain the motivation of the present work. The equilibrium theory of systems with infinitely many degrees of freedom has made many successes during the last decade, especially with regard to the interpretation of phase-transitions. It is tempting then to try to get some rigourous results in the domain of non-equilibrium theory. We quote here some typical problems which should be investigated on a rigourous basis:

1) Ergodic properties of thermodynamic equilibrium.

2) Existence of transport coefficients and their non-analytic behaviour at low density.

3) Irreversibility principle and H-theorem.

Unlike the third problem which lies in the field on non-equilibrium theory, the first two concern only dynamical properties in thermo-

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