Commun. math. Phys. 29, 89—104 (1973)

Infinite Volume Limits of the Canonical Free Bose Gas States on the Weyl Algebra

John T. Cannon
Rockefeller University, New York, USA

Received April 28; in revised form August 1, 1972

Abstract. It was shown by Araki and Woods that the infinite free Bose gas can be described by states on the Weyl algebra; they conjectured a certain family of states parameterized by temperature and density to be the infinite volume limit of the Gibbs canonical states. We show here that this conjecture is correct. We show that the volume dependent canonical states are equicontinuous in the density by a detailed calculation and a combinatorial result that gives cancellations. This allows us to develop a method of Kac that connects the canonical states explicitly with the grand canonical states which are more easily controlled in the infinite volume limit.

In 1963 Araki and Woods [1] showed that the theory of states on the Weyl algebra is a natural "quantum mechanics of infinitely many degrees of freedom" appropriate for the infinitely extended Bose gas. In this context, they conjectured a simple expression for the equilibrium state of the infinite free Bose gas at arbitrary temperature and density. In particular, their expression shows clearly the presence of the Einstein condensate above critical density. But the reasons for their conjecture were still based on the usual pre-quantum mechanical arguments of Einstein, which should be extraneous because the Gibbs canonical and grand canonical states of the finite Bose gas should simply converge in the infinite volume limit. So in the present note we give a direct proof that the Gibbs states do converge in the infinite volume limit. The canonical states converge to the state conjectured by Araki and Woods.

The main idea for the proof of convergence of the canonical states is due to Kac [2]. The infinite volume limit of the grand canonical states is easily calculated and the grand canonical state is a linear combination of canonical states at different densities. Kac showed that the coefficients of this linear combination converge to a simple distribution (in the infinite volume limit) which can be used to calculate canonical expectations from grand canonical expectations. Kac's work leaves open the technical problem of showing convergence of the canonical states themselves, but proves convergence of a wide class of linear combinations. In the present note, a slightly wider class of linear combinations is proved