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Plane Symmetric Self-Gravitating Fluids with Pressure Equal to Energy Density

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Abstract. Solutions of the Cauchy problem associated with the Einstein field equations which satisfy general initial conditions are obtained under the assumptions that (1) the source of the gravitational field is a perfect fluid with pressure, p, equal to energy density, w, and (2) the space-time admits the three parameter group of motions of the Euclidean plane, that is, the space-time is plane symmetric. The results apply to the situation where the source of the gravitational field is a massless scalar field since such a source has the same stress-energy tensor as an irrotational fluid with $p = w$. The relation between characteristic coordinates and comoving ones is discussed and used to interpret a number of special solutions. A solution involving a shock wave is discussed.

1. Introduction

The Einstein field equations for a self-gravitating perfect fluid with rest energy density w, pressure p and four-velocity u^μ may be written as

$$G^{\mu\nu} = R^{\mu\nu} - \tfrac{1}{2} g^{\mu\nu} R = -T^{\mu\nu} = -[(w+p)\, u^\mu u^\nu - p g^{\mu\nu}] \tag{1.1}$$

where

$$u^\mu u_\mu = 1 \tag{1.2}$$

and the units are chosen so that the velocity of light $c = 1$ and Newton's constant of gravitation $G = 1/8\pi$. As is well-known, Eqs. (1.1) must be supplemented by an additional requirement on the motion of the fluid. This may be taken to be an equation of state of the form

$$p = p(w) . \tag{1.3}$$

The velocity of sound in the fluid is then given by

$$a^2 = \frac{dp}{dw} . \tag{1.4}$$

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