Commun. math. Phys. 29, 43—47 (1973)

Tidal Accelerations in General Relativity

John Stewart and Martin Walker

Max-Planck-Institut für Physik und Astrophysik
München, Germany

Received July 3, 1972

Abstract. We derive the kinematical magnification of tidal accelerations which occurs for test bodies in highly relativistic orbits preferentially aligned with respect to repeated principal null rays of algebraically special spacetimes. Some examples of astrophysical importance are discussed. A general expression for computing tidal accelerations in the Newman-Penrose formalism is given.

1. Introduction

A problem of current interest in relativistic astrophysics is the behaviour of test bodies near black holes. This behaviour is likely to be most extreme when the speed v of the body approaches the local speed of light c. One question which arises in such a situation is whether tidal accelerations experienced by the body can become large enough to tear it apart. On the basis of previous work of Pirani [1] it might be thought that the magnitude of the tidal accelerations $T = O(\gamma^2)$ as $\gamma \to \infty$, where $\gamma = (1 - v^2 c^{-2})^{-1/2}$. In this case, a body whose velocity approached the local light velocity arbitrarily closely would presumably be torn apart. It is clearly of importance to determine under what conditions such a kinematical magnification of tidal accelerations does or does not take place.

In the next section, we present a theorem which gives sufficient conditions for the kinematical magnification mentioned above not to take place. The theorem applies only to situations in which the gravitational field (Weyl tensor) is algebraically special, but since there is strong evidence [2] to suggest that the exterior of a black hole can be described by one of the Kerr family of spacetimes (which are all of a particular algebraically special type), this restriction is not as severe as it may seem. In the third section we discuss briefly two cases in which the theorem does apply and one in which it does not, all of which are of potential astrophysical interest [3]. Finally in the fourth section we give an expression for the tidal acceleration in a completely general spacetime which may include both nongravitational fields and a cosmological term.