Equilibrium States of an Ising Ferromagnet in the Low Temperature Region

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Abstract. We investigate low temperature properties of an Ising ferromagnet when the nearest neighbour coupling constant is dominant. We show that all the translationally invariant equilibrium states are a superposition of only two extremal states.

Introduction

The existence of a phase transition has been proved for a large class of spin systems on a lattice [1, 2], but the determination of the number of pure phases cohexisting below the critical temperature is still an open problem. Only recently Gallavotti and Miracle Solè [3] have shown, for the Ising ferromagnet with only nearest neighbour interaction, that at low enough temperature all translationally invariant equilibrium states can be expressed as a superposition of only two extremal states. The aim of this paper is to extend their result to any finite range pair potential, provided that nearest neighbours interaction is negative, and "dominant".

We consider an Ising spin system, with a pair interaction, enclosed in a square box Λ with side L, on a two dimensional square lattice. The probability of a spin configuration in the box, given a spin configuration outside, is proportional to:

$$e^{-\beta E_{\Lambda}} = e^{-\beta \left\{\sum\limits_{(k,k') \in \Lambda} V(k-k') \sigma_{k} \sigma_{k'} + \sum\limits_{\substack{k \in \Lambda \\ k' \in \Lambda}} V(k-k') \sigma_{k} \sigma_{k'}\right\}}$$
(1)

¹ The meaning of this term is similar to that used in [1], see formula (2).