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On the Independence of the Thermodynamic Limit on the Boundary Conditions in Quantum Statistical Mechanics

N. Angelescu and G. Nenciu
Institute for Atomic Physics, Bucharest

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Abstract. The problem of the independence of the thermodynamic limit on the boundary conditions is considered in the framework of functional integration. For every domain and every boundary condition in a sufficiently large class a functional measure is constructed and the Feynman-Kac-like formula for the statistical operator written down. Making use of some volume-independent estimates for the Green function of the heat equation, the thermodynamic limit along convex domains for general boundary conditions is proved to exist and to be equal to that for Dirichlet conditions.

§ 1. Introduction

Essentially two methods have been used in quantum statistical mechanics for including the boundary conditions in the definition of local hamiltonians. One of them employs the connection between semi-bounded sesquilinear forms over a Hilbert space and semibounded self-adjoint operators [1, 2]. The other makes use of functional integration to write Feynman-Kac formulae for the semigroup of statistical operators $\exp(-\beta H)$, $\beta \geqq 0$. Initially devised to accomodate Dirichlet boundary conditions [3], this method was extended by Novikov [4], who considered the functional measure associated to the Wiener process in a parallelepipedic box with elastic reflecting walls and has thus been able to handle the case of Neumann boundary conditions for such domains

This paper is concerned with extending the second method for a larger class of domains and boundary conditions. In Section 2, functional measures suited for a class of boundary conditions are constructed, following the standard way [5] and using the appropriate Green function of the heat equation. Different properties of these measures, which follow from local estimates of the Green function, and the relation with the measures used by Ginibre [3] and Novikov [4] is established. In Section 3, the Feynman-Kac formula is written down and the equality of the thermodynamical limits for the whole class of boundary conditions