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Duality for Free Bose Fields

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Abstract. An elementary proof of Araki's duality theorem for free fields is presented. The theorem says that for a certain class of regions O in Minkowski space, the commutant of $\mathfrak{A}(O)$, the von Neumann algebra generated by all observables belonging to measurements within O, is exactly $\mathfrak{A}(O')$, where O' is the spacelike separated complement of O.

1. Introduction

In the algebraic approach to quantum field theory the principle of locality is expressed in the following way. There is a one to one correspondence between regions O in space-time and von Neumann algebras $\mathfrak{A}(O)$, the local algebras. Let $\mathfrak{A}(O)'$ denote the commuting algebra of $\mathfrak{A}(O)$ and let O' be the set of points which are spacelike separated from O. Then locality means that

$$\mathfrak{A}(O') \subset \mathfrak{A}(O)' .$$

An assumption stronger than locality is

$$\mathfrak{A}(O') = \mathfrak{A}(O)' .$$

This relation is called duality.

In recent investigations of Doplicher, Haag, and Roberts [6][1] on the connections between gauge group, superselection sectors and irreducible representations of the observable algebra, duality plays a crucial role. The importance of the duality relation in that work makes it desirable to know whether duality holds in any quantum field theory model. Araki [1, 2] has shown that in a free field theory duality holds; his proof seems to be rather complicated. The purpose of this paper is to show that with some modifications it can be simplified.

It should be mentioned that there is an elegant proof of duality for free fields in a publication by Dell'Antonio [5]. However the methods used there – von Neumann's infinite tensor products of Hilbert spaces –

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[1] Résumés of that work can be found in Ref. [7] and [9].