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Fields, Statistics and Non-Abelian Gauge Groups

Sergio Doplicher

Istituto Nazionale di Fisica Nucleare – Sezione di Roma
Istituto di Fisica "G. Marconi", Università di Roma

John E. Roberts

II. Institut für Theoretische Physik der Universität Hamburg

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Abstract. We examine field theories with a compact group $\mathscr{G}$ of exact internal gauge symmetries so that the superselection sectors are labelled by the inequivalent irreducible representations of $\mathscr{G}$. A particle in one of these sectors obeys a parastatistics of order d if and only if the corresponding representation of $\mathscr{G}$ is d-dimensional. The correspondence between representations of the observable algebra and representations of $\mathscr{G}$ extends to a mapping of the intertwining operators for these representations preserving linearity, tensor products and conjugation. Although we assume no explicit commutation property between fields, the commutation relations of fields of the same irreducible tensor character under $\mathscr{G}$ at spacelike separations are largely determined by the statistics parameter of the corresponding sector. For fields of conjugate irreducible tensor character the observable part of the commutator (anticommutator) vanishes at spacelike separations if the corresponding sector has para-Bose (para-Fermi) statistics.

1. Introduction

In studying the superselection structure of elementary particle physics one may distinguish the class of simple sectors. A simple sector is a superselection sector whose statistics is ordinary Bose or Fermi statistics. The set of simple sectors has the structure of a discrete Abelian group $\hat{\mathscr{G}}$ and one knows [1] that one can describe the sectors in a manner familiar from field theory by introducing unobservable quantities. Thus there is a field algebra $\mathfrak{F}$ made up of Bose and Fermi fields and a gauge group $\mathscr{G}$, the dual group of $\hat{\mathscr{G}}$, which allows us to recover the observable algebra as the gauge-invariant part of $\mathfrak{F}$.

We recall that $\mathfrak{F}$ is generated by the observable algebra together with a certain group of unitaries from $\mathfrak{F}$ called the field group. It was thus enough to construct a field group as a abstract group and this turned out to be equivalent to a standard problem in the theory of group extensions.

If we now consider, more generally, the set of finite sectors defined in [2] and therefore seek to include the sectors with parastatistics there is every reason to believe that a similar result should hold. Indeed, we