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Inequalities for Higher Order Ising Spins and for Continuum Fluids*

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Abstract. The recently derived Fortuin, Kasteleyn and Ginibre (FKG) inequalities for lattice gasses are investigated for higher order Ising spin systems and multi-component lattice gasses. Conditions are given for the validity of the FKG inequalities for higher order spin systems with Hamiltonians of the form used recently as models for various physical systems, e.g. $He^3 - He^4$ mixtures. We also investigate various inequalities for binary lattice gases and show how these can be carried over to continuum systems.

I. Introduction

In recent years a number of physical phenomena have been studied where systems consisting of Ising type particles of spin one, $S_i = 2, 0, -2$, or higher spin have been used as models. Some examples of such phenomena are a) the phase transitions of $U0_2$ [1] and $DyV0_4$ [2], b) annealed alloys of magnetic and non-magnetic atoms [3], c) the separation of components in a classical mixture [4, 5], and d) the λ-transition and phase separation in $He^3 - He^4$ mixtures [6]. Higher order spin systems have also been investigated in order to gain insight into the general nature and existence of phase transitions [7].

An interesting question in these investigations is the extent to which the many results known for simple spin $\frac{1}{2}$ Ising systems, $S_i = \pm 1$, with ferromagnetic interactions remain valid for these higher spin systems. This question was partly answered by Griffiths [7] who showed that every higher order Ising spin system can be "mapped" into some spin $\frac{1}{2}$ Ising systems. Higher spin systems with *purely* ferromagnetic interactions were reduced to spin $\frac{1}{2}$ systems with the same property. This showed immediately that the Griffiths [8], Kelly and Sherman [9] (GKS) inequalities for spin $\frac{1}{2}$ Ising systems with ferromagnetic interactions re-

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