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The Finite Group Velocity of Quantum Spin Systems

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Abstract. It is shown that if Φ is a finite range interaction of a quantum spin system, τ_t^{Φ} the associated group of time translations, τ_x the group of space translations, and A, B local observables, then

$$\lim_{\substack{|t|\to\infty \\ |x|>v|t|}} \| [\tau_t^{\Phi}\tau_x(A), B] \| \, e^{\mu(v)t} = 0$$

whenever v is sufficiently large ($v > V_{\Phi}$) where $\mu(v) > 0$. The physical content of the statement is that information can propagate in the system only with a finite group velocity.

1. Introduction

In [2] it was demonstrated that for a large class of translationally invariant interactions, time translations of quantum spin systems can be defined as automorphisms of a C^*-algebra, $\mathcal{A}$, of quasi-local observables, i.e. the abstract algebra generated by the spin operators. This should allow one to discuss features of the dynamical propagation of physical effects in an algebraic manner independent of the state of the system, i.e. independent of the kinematical data. It is expected that this propagation has many features in common with the propagation of waves in continuous matter and the point of this paper is to demonstrate such a feature, namely a finite bound for the group velocity of a system with finite range interaction. This result is obtained by a simple estimation derived from the equations of motion and it is possible that more detailed estimations would give more precise information of the form of spin-wave propagation. We briefly discuss this possibility at the end of Section 3.

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