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Thermodynamic Functions for Fermions with Gravostatic and Electrostatic Interactions*

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Abstract. The energy as function of entropy and the free energy as function of temperature is calculated rigorously for nonrelativistic fermions with

$$\frac{e' e'' - \kappa M' M''}{|\vec{x}' - \vec{x}''|}$$

interactions. It is shown that in the appropriate thermodynamic limit the corresponding Thomas-Fermi equation becomes exact.

I. Introduction

In this paper we evaluate the thermodynamic functions for matter in the limit of many particles. By "matter" we mean nonrelativistic fermions interacting with Coulomb- and Newton potentials. For definiteness we consider two species of equally many fermions (electrons and protons, say) with masses M_1, M_2 and opposite charges $\pm e$. The reason why the thermodynamic functions can be calculated is that for a large number N of particle pairs the system is compressed by gravitation to such an extent that the Thomas-Fermi equation becomes exact. The present results generalize those of a previous paper [1] in two respects:

a) Electrostatic forces are also included. Our results show that for $N \to \infty$ they cannot prevent the compression of the system. This is in accordance with the folklore that for

$$N > \left(\frac{e^2}{\kappa M_p^2}\right)^{3/2} \sim 10^{54}$$

atoms are squashed and one obtains a high-density plasma. For the ground state this fact has been demonstrated by Levy-Leblond [2].

b) We also show that if one calculates the energy as function of entropy in the microcanonical ensemble the relevant Thomas-Fermi

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