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Determination of the Scattering Amplitude from the Differential Cross-Section and Unitarity*

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Abstract. When the differential cross-section for spin-zero elastic scattering is given, the elastic unitarity condition constitutes a nonlinear integral equation for the phase of the scattering amplitude. Existence and uniquences theorems for solutions of the equation were obtained by Newton and Martin. Some improvements of the Newton-Martin results on uniqueness and iterative construction of solutions are obtained. Certain details of rigour in the applications of Schauder's theorem by Newton and by Martin are supplied. The case of inelastic spin-zero scattering is treated by adding a term to the unitarity condition to account for absorption. It is shown that in the inelastic region one may have infinitely many different scattering amplitudes with a given differential cross-section. This result is potentially important in phase-shift analysis, since it means that there is a "continuum ambiguity" in the determination of phases and elasticities from scattering data.

I. Introduction

A fundamental question of scattering theory is this: to what extent is the scattering amplitude determined when the differential cross-section is given and the unitarity condition is imposed? The problem is interesting in connection with phase-shift analysis, as well as in epistomological questions of quantum theory. For the case of spin-zero elastic scattering, a partial answer was provided by Crichton [1], Newton [2], Martin [3], and others [4]. Newton and Martin were the first to obtain sufficient conditions for existence and uniqueness of an elastic amplitude with a given cross-section.

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