Commun. math. Phys. 28, 123—132 (1972)


# Symmetry of the Physical Probability Function Implies Modularity of the Lattice of Decision Effects*

Günter Dähn

Mathematisches Institut der Universität Tübingen

Received April 18, 1972

**Abstract.** This paper states two equivalent conditions from which modularity of the lattice $G$ of decision effects $E$ can be derived. It may be seen as a supplement to Ludwig's approach [5] to an axiomatic foundation of physical theories. As a consequence of these conditions every filter $\boldsymbol{T}_E$ is a self-adjoint projector on the Hilbert space $B'$ generated by the decision effects.

## I. The Construction of a Canonical Linear Order Isomorphism

The mathematical symbols and definitions used in the sequel are taken from [1, 2] without further explications. Theorem 4 in [1] states the existence of a bijection $\underline{J}$ between the set of all atoms $P$ of $G$ and the set $A(W)$ of all atoms $K_1(P)=\{V_P\}$ of $W$. This first part of our paper is concerned with the possibility of extending that bijection $\underline{J}$ to the whole of $B'$ such that its extension $J$

(i) becomes a *canonical* linear isomorphism between $B'$ and $B$ and

(ii) preserves *order* in both directions.

*Remark 1.* Let us remember a well-known fact from linear algebra: Given two $\boldsymbol{R}$-vector spaces $B_1$, $B_2$ and any linearly independent set $S \subset B_1, S \neq \emptyset$:

(a) if $\underline{h}: S \to B_2$ is a map, then there exists a unique linear extension $h: \mathrm{lin}_{\boldsymbol{R}} S \to B_2$

(b) the linear extension $h$ of $\underline{h}$ is injective iff

($b_1$) $\underline{h}$ is injective and

($b_2$) $\underline{h}[S]$ is linearly independent.

Of course, $B'$, $B$ having the same finite dimension, say $N$, they are isomorphic and the isomorphism depends, in general, on the basis chosen. The very problem here is to point out a *canonical* isomorphism $J$ between $B'$ and $B$ such that

$$J\,|\,A(G)=\underline{J} \quad \text{and} \quad J^{-1}\,|\,A(W)=\underline{J}^{-1}\,.$$

---

* This paper extends a final report presented to and supported by the Deutsche Forschungsgemeinschaft.