Commun. math. Phys. 28, 69—86 (1972)

The Thermodynamic Limit for an Imperfect Boson Gas

E. B. Davies
Mathematical Institute Oxford, England

Received April 7, 1972

Abstract. We give a rigorous treatment in the infinite volume limit of a model Hamiltonian representing an imperfect Boson gas. In particular we obtain the exact expression for the mean particle density in the infinite volume limit as a function of the chemical potential, and show that the density function has a singularity at the critical density for Bose-Einstein condensation. We prove that, unlike the ideal Boson gas, the imperfect Boson gas has the same behaviour in the infinite volume limit for the grand canonical ensemble as for the canonical ensemble, and is moreover stable under small perturbations. We finally exhibit the possibility of ordinary condensation and prove that a system in an intermediate situation between two pure phases consists of a simple mixture of the two phases involved.

§ 1. Introduction and Notation

We let $\Lambda \subseteqq \mathscr{R}^3$ be an open region of unit volume with smooth boundary and for $L \geqq 1$ let

$$\Lambda_L = \{Lx : x \in \Lambda\}\,, \tag{1.1}$$

$$\mathscr{H}^L = L^2(\Lambda_L) \tag{1.2}$$

and let $\mathscr{F}^L$ be the symmetric Fock space constructed from $\mathscr{H}^L$. We let S^L be a self-adjoint Hamiltonian on $\mathscr{H}^L$ with discrete spectrum and eigenvalues

$$0 = L^{-2}E_0 < L^{-2}E_1 \leqq L^{-2}E_2 \leqq \cdots \tag{1.3}$$

counted according to multiplicity. We suppose certain asymptotic conditions on the growth of the eigenvalues which are satisfied in the case $S^L = -\frac{1}{2}\Delta$, and denote by H_0^L the free Hamiltonian on $\mathscr{F}^L$ constructed from S^L in the usual manner.

The Hamiltonian that we consider in this paper is

$$H_\mu^L = H_0^L - \mu N^L + L^3 f(N^L/L^3) \tag{1.4}$$

where N^L is the number operator on $\mathscr{F}^L$ and f is a continuously differentiable function on $[0, \infty)$ satisfying $f(0) = 0$ and

$$\lim_{x \to \infty} f'(x) = +\infty\,.$$