Commun. math. Phys. 28, 47–67 (1972)

Feynman's Path Integral

Definition Without Limiting Procedure

CÉCILE MORETTE DEWITT*
Department of Astronomy, University of Texas at Austin, USA

Received February 11, 1972; in revised form May 25, 1972

Abstract. Feynman's integral is defined with respect to a pseudomeasure on the space of paths: for instance, let $\mathscr{C}$ be the space of paths $q : T \subset \mathbb{R} \to$ configuration space of the system, let $\mathscr{C}'$ be the topological dual of $\mathscr{C}$; then Feynman's integral for a particle of mass m in a potential V can be written

$$\int_{\mathscr{C}} \exp(i S_{\text{int}}(q)/\hbar)\, dw(\sqrt{m}\, q)$$

where

$$S_{\text{int}}(q) = \int_T V(q(t))\, dt$$

and where dw is a pseudomeasure whose Fourier transform is defined by

$$\mathscr{F} w(\mu) = \exp(-i W(\mu)/2) = \exp\left(-\frac{i}{2} \int_T \int_T \inf(t, t')\, d\mu(t)\, d\mu(t')\right)$$

for $\mu \in \mathscr{C}'$. Pseudomeasures are discussed; several integrals with respect to pseudomeasures are computed.

I. Introduction

The lucid and powerful formalism of quantum mechanics proposed by Feynman [1] has been plagued by the limiting procedure involved in the original definition of Feynman's integral. We propose here a definition which does not rest on a limiting procedure, we show the connection between both definitions of Feynman's integral and we compute several integrals.

Feynman's formalism of quantum mechanics can be summarized in the following table:

1. Quantum experiments $\Rightarrow K(B; A) = \int_X \exp(iF(q)) \ldots$
2. Classical limit of quantum systems $\Rightarrow K(B; A) = \int_X \exp(iS(q)/\hbar) \ldots$

---

* This work has been supported in part by a NATO Research Grant and by a National Science Foundation grant [GP-15184; GP-20033].