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The Einstein Evolution Equations as a First-Order Quasi-Linear Symmetric Hyperbolic System, I

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Abstract. A systematic presentation of the quasi-linear first order symmetric hyperbolic systems of Friedrichs is presented. A number of sharp regularity and smoothness properties of the solutions are obtained. The present paper is devoted to the case of $\mathbf{R}^n$ with suitable asymptotic conditions imposed. As an example, we apply this theory to give new proofs of the existence and uniqueness theorems for the Einstein equations in general relativity, due to Choquet-Bruhat and Lichnerowicz. These new proofs using *first order* techniques are considerably simplier than the classical proofs based on *second order* techniques. Our existence results are as sharp as had been previously known, and our uniqueness results improve by one degree of differentiability those previously existing in the literature.

§ 0. Introduction

Part of the folklore of mathematics is that the Friedrich's theory of symmetric hyperbolic systems extends to the quasi-linear case. Our original motivation for looking at these systems came from the fact that it is possible to reduce the Einstein system studied by Choquet-Bruhat and Lichnerowicz [3, 4, 33] to a *first order* symmetric hyperbolic system. The techniques these authors used are based on the *second order* theory of Leray [32] as improved by Dionne [16].

However we needed a version of the symmetric hyperbolic theory with sharper differentiability properties than previously existed in the literature. The basic theory is presented in §§ 1, 2 below. We consider the equations in $\boldsymbol{R}^n$ with asymptotic conditions imposed. Presumably similar results are true for bounded regions with suitable boundary conditions. One could also argue locally in $\boldsymbol{R}^n$ and use domain of dependence arguments; cf. Fischer-Marsden [21], Wilcox [42]. Part II will deal with the theory on manifolds.

This theory is complicated in its technical details by two facts. First, differentiability properties of the coefficients complicate the proof that the solutions are just as differentiable as the initial data in the Sobolev class H^s. Second, we want this value of s to be the best possible, $s > n/2 + 1$.

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