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Diffusion for Weakly Coupled Quantum Oscillators

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Abstract. We construct a simple model which exhibits some of the properties discussed by van Hove in his study of the Pauli master equation. The model consists of an infinite chain of quantum oscillators which are coupled so that the interaction Hamiltonian is quadratic. We suppose the chain is in equilibrium at an inverse temperature β and study the return to equilibrium when a chosen oscillator is given an arbitrary perturbation. We show that in the limit as the interaction becomes weaker and of longer range, the evolution of the chosen oscillator becomes a diffusion equation. Moreover we give an explicit example where the evolution of the chosen oscillator has the Markov property and where the Pauli master equation is exactly satisfied.

§ 1. Introduction

We consider a linear chain of quantised harmonic oscillators interacting by a quadratic Hamiltonian in very much the same spirit as Ford, Kac and Mazur [1]. However, instead of taking a finite number $(2N+1)$ of oscillators and going to the limit $N \to \infty$, we take an infinite number of oscillators from the beginning, with an interaction which has a cut-off depending on a parameter $\lambda > 0$, and go to the limit as $\lambda \to 0$.

We let the Hamiltonian of the system be

$$H_\lambda = H_0 + H_{I,\lambda} \tag{1.1}$$

where

$$H_0 = \tfrac{1}{2} \sum_{n=-\infty}^{\infty} :p_n^2 + \omega^2 q_n^2: \tag{1.2}$$

and

$$H_{I,\lambda} = \sum_{m,n} :\alpha_{m-n}^{(\lambda)} q_m q_n: \tag{1.3}$$

For the time being we suppose only that $a_m^{(\lambda)}$ are real coefficients satisfying

$$\sum_{m=-\infty}^{\infty} |a_m^{(\lambda)}| < \infty \tag{1.4}$$

for all $\lambda > 0$. The operators p_m and q_n are supposed to satisfy the commutation relations

$$[q_m, p_n] = i\delta_{mn}; \quad [q_m, q_n] = [p_m, p_n] = 0\,. \tag{1.5}$$