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The Dispersion of Gravitational Waves

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Abstract. A definition is given of a plane gravitational wave in a curved background space-time manifold. For a particular background metric, a dispersion relation for the waves is derived analogous to that satisfied by plane electromagnetic waves in a dilute plasma.

I. Introduction

Our purpose here is to discuss the dispersion of gravitational waves in vacuo. We give a definition of a plane wave in a curved background space-time manifold and for a particular background metric a dispersion relation for these waves is derived. We formulate the results in a general manner but because of certain ad hoc assumptions we are forced to make, we can offer no reason to believe that they are valid for any background metric other than that given as an example.

In comparison, we first give the electromagnetic case in Section II. This is a test of the validity of the wave ansatz. There can be no dispersion of electromagnetic radiation in the absence of charged matter.

II

In this section we shall discuss electromagnetic waves in a region of space-time in which there is no charged matter present. If there is neutral matter then we suppose that the dielectric constant and the permeability are both equal to one.

An electromagnetic wave is described by a Maxwell tensor of the form

$$\tilde{F}_{\mu\nu} = F_{\mu\nu} + \varepsilon A_{\mu\nu}\,, \tag{2.1}$$

where $F_{\mu\nu}(x^\sigma)$ describes an arbitrary given background electromagnetic field. ε is a dimensionless constant characterizing the order of magnitude of the amplitude of the wave and $A_{\mu\nu}$ is given by

$$A_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu\,, \tag{2.2}$$

where

$$A_\mu = A_\mu(x^\sigma, \omega\phi)\,, \tag{2.3}$$