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A Class of Solutions of Einstein's Equations which Admit a 3-Parameter Group of Isometries

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Abstract. The Plebański and Stachel and Goenner and Stachel lists of metrics which are solutions of Einstein's field equations, have two double eigenvalues and admit 3-parameter groups of isometries with 2-dimensional spacelike orbits are completed by the addition of metrics which result from the use of a more general metric form.

1. Introduction

Plebański and Stachel [1] state that they have carried out a complete classification of spherically-symmetric metrics whose Einstein tensors have two double eigenvalues. Goenner and Stachel [4] extend Ref. [1] to include the cases in which the two-curvature of the spacelike orbit may be zero or negative. However, as was pointed out by Takeno and Kitamura [2] and Goenner [3] the general form of the metric used by Plebański and Stachel and Goenner and Stachel

$$ds^2 = e^{\nu}(dx^0)^2 - e^{\lambda}dr^2 - r^2 d\omega^2 \tag{1}$$

where

$$d\omega^2 = d\theta^2 + \Sigma^2 d\phi^2$$

$$\text{and also where } \Sigma = \begin{cases} \sin\theta & \text{for positive two-curvature} \\ \sinh\theta & \text{for negative two-curvature} \\ 1 & \text{for zero two-curvature}. \end{cases}$$

in fact excludes one class of metrics. Here $\lambda = \lambda(x^0, r)$ and $\nu = \nu(x^0, r)$.

Goenner [3] extends Ref. [4] to include the class which had previously been omitted without, however, specifying the additional metrics involved. These metrics are listed in this paper.

From Goenner [3] the general form of a metric which admits a three parameter group of isometries with two-dimensional spacelike orbits (i.e. $G_3(2, s)$) is

$$ds^2 = 2G(u, v)\,du\,dv - M^2(u, v)(d\theta^2 + \Sigma^2 d\phi^2). \tag{2}$$