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On the Asymptotic Behavior of Wightman Functions in Space-Like Directions* **

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Abstract. The asymptotic behavior of the truncated vacuum expectation value of a product of N (unbounded) quasilocal operators, $F(x) = \langle Q_1(x_1) \ldots Q_N(x_N) \rangle_T$, is investigated for some of the separations space-like. It is shown that unless all clusters $\{x_{i_1}, \ldots, x_{i_j}\}$ are partially time-like (or light-like) separated from their complements $\{x_{i_{j+1}}, \ldots, x_{i_N}\}$, $F(x)$ decreases faster than any inverse power of the diameter of the set $\{x_1, \ldots, x_N\}$.

I. Introduction

The asymptotic behavior of the vacuum expectation value (VEV) of a product of field operators, $\langle 0 | \varphi(x_1) \ldots \varphi(x_N) | 0 \rangle$, has been studied by many authors [1–5] for some of the separations, $x_i - x_j$, space-like. Although rapid decrease of the truncated VEV (after smearing with rapidly decreasing test functions) has been proved for $x = (x_1, \ldots, x_N)$ in some regions of $\mathbb{R}^{4N}$, there does not seem to be any general statement of the space-like asymptotic behavior of this function available in the literature[1]. In this note we extend the method of Ruelle [3] to show fast decrease in a much larger region of $\mathbb{R}^{4N}$.

II. Definitions and Results

We consider a scalar Wightman field [7], $\varphi(x)$, and define the "quasi-local" operators

$$Q_i(0) = \int \left(\prod_{j=1}^{M_i} d^4 y_j \right) f_i(y_1, \ldots, y_{M_i}) \, \varphi(y_1) \ldots \varphi(y_{M_i}), \tag{1}$$

for $i = 1, \ldots, N$. Here $f_i \in \mathscr{S}$, the Schwartz space of infinitely differentiable functions which decrease (along with all derivatives) faster than any

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[1] We thank R. Haag for pointing out the work of H. Araki [6] whose results for the truncated VEV of *bounded* operators are essentially equivalent to our Theorem 1. We remark that Araki's proof does not generalize to unbounded operators.