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# The Field Algebra and Its Positive Linear Functionals

W. Wyss*
Department of Mathematics and Physics, University of Colorado, Boulder, Colorado

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**Abstract.** We show that positive linear functionals on the field algebra are necessarily continuous and can be represented by conical measures. Furthermore extension theorems for continuous linear functionals, defined on a subspace of the field algebra, to positive linear functionals are discussed.

## 1. Introduction

It is well known [1] that Wightman's axiomatic theory of quantized fields can be discussed in terms of a topological *-algebra $\mathfrak{A}$, called the field algebra, and its positive linear functionals that vanish on a certain subspace. The study of continuous linear functionals vanishing on this subspace is the content of the so called linear program in Quantum Field Theory. Our intention here is to learn as much as possible about positive continuous linear functionals on $\mathfrak{A}$. In Section 2 we study the field algebra, its hermitean part and the positive cone $K$. In Section 3 we find that a positive linear functional on $\mathfrak{A}$ is always continuous. Hence the continuity requirement in Wightman's axioms can be dropped. Section 4 shows that positive functionals are related to conical measures. In Section 5 we are interested in the following situation: given a continuous linear functional $T$ on a closed subspace $M \subset \mathfrak{A}$ and positive on $K \cap M$. Furthermore let $N$ be another closed subspace of $\mathfrak{A}$. Under what conditions does there exist an extension of $T$ to a positive linear functional on $\mathfrak{A}$, vanishing on $N$? We have necessary and sufficient conditions for this situation. Section 6 deals with some applications to Wightman's field theory.

## 2. The Field Algebra $\mathfrak{A}$, its Hermitean Part $\mathfrak{A}_0$ and the Positive Cone K

For our purposes the field algebra $\mathfrak{A}$ is modeled over $\mathfrak{S}(\mathbb{R}^{4n})$, the Laurent Schwartz test function space [2], as follows

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