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Renormalization of Non Polynomial Lagrangians in Jaffe's Class

H. EPSTEIN
I.H.E.S., Bures-sur Yvette

V. GLASER
CERN, Geneva

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Abstract. It is shown how a renormalized perturbation series can be defined for a theory with strictly local, non-polynomial, interacting Lagrangian

$$\mathscr{L}(x) = \sum_{r=0}^{\infty} t_r \frac{:A(x)^r:}{r!}$$

so as to preserve locality at every order.

1. Introduction

In Ref. [1] an inductive construction of the perturbation series for polynomial Lagrangians was given. It was shown that, given a Wick polynomial $\mathscr{L}(x)$ in a free field $A(x)$ one can construct (by induction on n) the chronological products

$$T(x_1, \ldots, x_n) = T(\mathscr{L}(x_1) \ldots \mathscr{L}(x_n))$$

which generate the perturbation series of a theory whose interaction Lagrangian reduces to $\mathscr{L}(x)$ in the first order. In this note we show how to extend this method to the case when $\mathscr{L}(x)$ is no longer a Wick polynomial but an entire function in the free field, denoted

$$\mathscr{L}(x) = \sum_{r=0}^{\infty} t_r \frac{:A(x)^r:}{r!}. \tag{1}$$

Such an entire function is still a strictly localized field in the sense of Jaffe [2] provided the coefficients t_r do not grow too fast; in fact we shall restrict ourselves to a special class of theories in which

$$|t_r| < EM^r r^{\lambda r} \tag{2}$$