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Exact Solution of the Dirac Equation with a Central Potential

E. J. KANELLOPOULOS, TH. V. KANELLOPOULOS, and K. WILDERMUTH
Institut für Theoretische Physik der Universität Tübingen

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Abstract. The exact solution of the Dirac equation with a central potential, in the semi-relativistic approximation, is derived and formulae for phase shifts and eigenvalue equations are given.

Introduction

The integro-iteration method, introduced in Ref. [1] is applied to the solution of the Dirac's coupled radial equations. The solutions are obtained in a form similar to that of the Schrödinger equation [2], i.e., in simple series which converge strongly when the following restrictions are imposed on the potential $V(r)$:

$$V_{r\to 0}(r) \rightsquigarrow r^{-\beta}\ \beta \leqq 1 \tag{1a}$$

and

$$\int_a^\infty V(r)\,dr < \infty \quad \text{for} \quad 0 < a < \infty . \tag{1b}$$

Condition (1 b) excludes the Coulomb potential, but in this case the solutions are already known [3, 4]. On the other hand in cases with a screened or modified Coulomb potential [5] the method is applicable and one can get results to any desired accuracy.

I. Formulation

In semi-relativistic approximation the Dirac equation with central potential, after separation of the angular part, [3], is reduced to a system of two coupled radial equations [5];

$$\begin{aligned} (E+V+m)F_\nu + \frac{dG_\nu}{dr} - \frac{\nu}{r} G_\nu &= 0 \\ -(E+V-m)G_\nu + \frac{dF_\nu}{dr} + \frac{\nu+2}{r} F_\nu &= 0 . \end{aligned} \tag{2}$$