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Asymptotic Completeness for Multi-Particle Schroedinger Hamiltonians with Weak Potentials

RAFAEL JOSE IORIO, JR. and MICHAEL O'CARROLL
Departamento de Matemática, Pontifícia Universidade Católica, Rio de Janeiro, Brasil

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Abstract. We show that the non-relativistic quantum mechanical n-body Hamiltonians $T(k) = T + kV$ and T, the free particle Hamiltonian, are unitarily equivalent in the center of mass system, i.e., $T(k) = W_{\pm}(k)\, T W_{\pm}(k)^{-1}$ for k sufficiently small and real. $V = \sum_i V_i$, a sum of $\frac{n(n-1)}{2}$ real pair potentials, V_i, depending on the relative coordinate $x_i \in R^3$ of the pair i, where V_i is required to behave like $|x_i|^{-2-\varepsilon}$ as $|x_i| \to \infty$ and like $|x_i|^{-2+\varepsilon}$ as $|x_i| \to 0$. $T(k)$ is the self-adjoint operator associated with the form sum $T + kV$. There are no smoothness requirements imposed on the V_i. Furthermore $W_{\pm}(k) = s\text{-}\lim_{t \to \pm\infty} e^{iT(k)t} e^{-iTt}$ are the wave operators of time dependent scattering theory and are unitary. This result gives a quantitative form of the intuitive argument based on the Heisenberg uncertainty principle that a certain minimum potential well depth and range is needed before a bound state can be formed. This is the best possible long range behavior in the sense that if $kV_i \leqq C_i |x_i|^{-b}$, $0 < b \leqq 2$ for $|x_i| > R_i (0 < R_i < \infty)$ and all C_i are negative then $T(k)$ has discrete eigenvalues and $W_{\pm}(k)$ are not unitary.

0. Introduction

In this article we treat the scattering and spectral problem for an n-body system in non-relativistic quantum mechanics with weak potentials. We show that the method of Kato [1] used to show asymptotic completeness and unitarity of the wave operators for weak potentials in the two-body case can be applied to obtain similar results in the n-body case. More precisely we show that in the center of mass system Hilbert space $H = L^2(R^{3n-3})$ the self-adjoint operators $T(k) = T + kV$ (the self-adjoint operator associated with a form sum) and T (the free particle Hamiltonian) are unitarily equivalent for sufficiently small, real k. The potential $V = \sum_i V_i$ is a sum of pair potentials, V_i, which are real-valued measurable functions depending on the relative coordinates $x_i \in R^3$ of the pair i. Writing

$$A_i = |V_i|^{1/2}, \qquad B_i = (\operatorname{sign} V_i) A_i,$$

the result follows from the crucial fact that the operators $A_i(T-z)^{-1} B_j^*$ admit bounded analytic extensions for $\operatorname{Im} z \neq 0$, the bound being in-