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Exact Gravitational Field of the Infinitely Long Rotating Hollow Cylinder

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Abstract. The vacuum line element inside an infinitely long rotating hollow cylinder is the usual flat space line element. It is fitted in a most general way to the general cylindrical vacuum field outside at the singular hypersurface $R_0 = \text{const}$, representing the infinitely thin hollow cylinder. With the use of the jump conditions at $R_0 = \text{const}$ the surface densities τ_μ^ν, of which the energy-momentum-stress tensor τ_μ^ν of the shell consists, are calculated. The physical properties of the cylinder, as derived from the eigenvalues and -vectors of τ_μ^ν, and the generated gravitational field are discussed in full detail.

1. Introduction

Recently we have shown [1] (in the following cited as I), that the general stationary cylindrical vacuum field, found by Davies and Caplan [2] is static, whereafter, it is identical with Levi-Civitas general static solution [3]. Hence any *stationary* (rotating) cylindrical matter distributions generate a *static* cylindrical vacuum field. As far as we know the only rigorously treated example for this class of matter distributions is the rotating cylinder of Van Stockum [4], consisting of incoherent matter.

In this paper we present the general solution for the uniformly rotating infinitely thin hollow cylinder. The general-relativistic procedure of constructing the gravitational field of such surface distributions has been given by Lanczos [5], Israel [6], Treder [7] *et al.* The main results, which we shall need in this paper, are: Choosing natural (Gaussian) coordinates in which the metric tensor is continuous across the (singular) hypersurface $x_1 = a = \text{const}$, we get the line-element in the form

$$ds^2 = -dx^{1^2} + g_{ik} dx^i dx^k \qquad (i, k = 2, 3, 4). \tag{1.1}$$

The energy-momentum-stress tensor T_μ^ν has the surface-density structure[1]

$$T_\mu^\nu = \tau_\mu^\nu \, \delta(x_1 - a). \tag{1.2}$$

According to the definition of the δ-function Einstein's field equations of gravitation

$$R_{\mu\nu} = -(T_{\mu\nu} - \tfrac{1}{2} T g_{\mu\nu}) \tag{1.3}$$

[1] Greek indices run from 1–4, latin indices (except i, k) from 1–3.