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On the Geometrical Interpretation of the Harmonic Analysis of the Scattering Amplitude

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Abstract. In this paper we intend to analyze the geometry underlying the various representations of the relativistic scattering amplitudes. More precisely we consider the direct-channel expansion, its euclidean contraction and the crossed-channel representation. In all these representations one can distinguish the factors which express the dynamics from those which reflect the symmetry; starting from the latter, one can try a geometrical interpretation of the harmonic analysis of the scattering amplitude on the Poincaré group.

I. Introduction

In the last decade, many authors [1–5] contributed in making clear the role played in particle physics by the harmonic analysis of the scattering amplitude on the Poincaré group. Consider the elastic scattering of two scalar particles of equal mass m and denote with $T(s, t)$ the scattering amplitude, where s and t are the usual Mandelstam variables; i.e. s is the energy squared and t the momentum-transfer squared. In a paper which appeared in 1962 Joos [5] considered the relativistic phase-shift analysis in the so-called s-channel. In this case one takes $s > 0$ fixed and the scattering amplitude is decomposed in functions of $z = \cos\vartheta = 1 + \frac{2t}{s - 4m^2}$; i.e. the cosine of the scattering angle in the center of mass system. These functions are the Legendre polynomials and the little group is the rotation group. However one can also invert the situation and take $t < 0$ fixed and decompose the scattering amplitude in functions of s obtaining in this way the so-called t-channel (or crossed-channel) phase-shift analysis. In the latter case, instead of the center of mass system the convenient system is the brick-wall system [4] and the little group is the non-compact group $SO(2, 1)$. One of the great advantages of these decompositions is that the scattering amplitude is thereby separated into its dynamical part, contained in the partial-