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Einstein Algebras

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Abstract. An approach to quantization of general relatively using a reformulation of the classical theory in which the events of space-time play essentially no role is discussed.

1. Introduction

An event in physical space-time becomes, when idealized in general relativity, a point of some four-dimensional manifold. Such an idealization is natural in classical physics because, in this case, it is possible in principle to make arbitrarily precise observations whose effects on the system observed can be made arbitrarily small. The fundamental role of the events in the mathematical formalism of general relativity reflects this potential for precise measurement. In quantum theory, on the other hand, the influence of the measuring apparatus on the system being observed cannot, even in principle, be made arbitrarily small. Furthermore, in a quantum theory of the gravitational field, one would expect the metric itself to be subject to quantum fluctuations. But the metric is the primary tool for isolating individual events [1]. Thus, it is perhaps reasonable to expect that, in a quantum theory of gravitation, the mathematical formalism will, at some point, suggest a "smearing out of events".

In the various approaches to quantization of the gravitational field [2–7], one retains, at least in some form, the events of space-time. It is presumably intended that these events will lose their significance in the final theory. Although it might seem more natural to adopt an approach in which events play a secondary role from the beginning, this turns out to be difficult, for the set of events, i.e., the underlying manifold, is central in the standard treatments of classical general relativity.

The purpose of this paper is to point out that, by a judicious choice of definitions, the entire content of general relativity can be so formulated that the underlying manifold plays practically no role. This version of the theory may offer a convenient starting point for quantization. The smearing out of events, which is to be expected in the quantum