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On Quantum Systems in Thermal Contact

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Abstract. It is shown that under rather general conditions two K.M.S. states ω_1 and ω_2 of systems S_1 and S_2, respectively, can be simultaneously extended to a K.M.S. state ω of a system composed of S_1 and S_2, provided both systems have equal temperatures. This result gives further support to the conjecture that K.M.S. states are equilibrium states. In the second part, a model of thermal coupling is constructed which satisfies the assumptions of the first part, thereby showing that the result is also valid in the interesting case of systems S_1 and S_2 in thermal contact.

I. Introduction

During the past three years, there appeared an increasing number of papers on the Kubo-Martin-Schwinger boundary condition strongly supporting the assumption that a state ω of a general quantum statistical system is an equilibrium state if and only if ω satisfies the K.M.S. condition [1–6]. It is understood that the systems under consideration do not contain adiabatically closed subsystems of different temperatures. There are proofs of this conjecture for special cases [1–4], but, to my knowledge, no general proof exists. This paper is intended to give further support to the above conjecture.

If we consider two systems S_1 and S_2 described by the respective algebras of observables $\mathfrak{A}_1, \mathfrak{A}_2$ and the states ω_1 and ω_2 which satisfy the K.M.S. condition, we may ask whether ω_1 and ω_2 have a common extension to a K.M.S. state ω of a system composed by S_1 and S_2, provided S_1 and S_2 have equal temperatures. If K.M.S. states are equilibrium states, we expect this to be true. In the next section, we shall give a proof of this conjecture using rather general assumptions. In Section III, we shall exhibit a model of thermal coupling of two finite quantum systems. Finally, we shall deal with the infinite volume limit and show that the systems are continuous in the coupling parameter, and, in addition, we shall argue that the states of the infinite systems are essentially the same as those of the uncoupled systems if the temperatures are equal. The results of Sections III and IV are intended to show that the assumptions of Section II are reasonable even in the presence of thermal coupling.