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Higher Order Perturbation Theory for Exponential Lagrangians: Third Order*

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Abstract. We define the vacuum expectation value of the time-ordered product of three exponentials of free massless fields as a continuous linear functional over a suitable test function space using minimal singularity as a criterion.

I. Introduction

The present paper is an extension of an earlier work [1] devoted to the analysis of the structure of exponential interactions as given by the Lagrangian $\mathscr{L}_{\text{int}}(f\phi)$

$$\mathscr{L}_{\text{int}}(f\phi(x)) = :e^{f\phi(x)} - 1: = L_{\text{int}}(x) \tag{1}$$

where ϕ is a free scalar field of mass m.

In [1] we discussed the second order contribution to the Green's functions in an expansion in powers of $\mathscr{L}_{\text{int}}(f\phi)$. To achieve uniqueness we introduced a minimality principle. We argued that with the least singular choice of the time-ordered product $TL_{\text{int}}(x_1)\,L_{\text{int}}(x_2)$ the Green's functions correspond most closely to the given classical Lagrangian (in second order).

Here we go one step beyond the results of Ref. [1] and show that the minimality principle can be generalized to third order, at least for the case of a massless field. The generalized minimality principle leads to a unique, least singular definition of the time-ordered product $TL_{\text{int}}(x_1)\ldots L_{\text{int}}(x_3)$. Because of the simple relation between time- and normal-ordered products of exponential Lagrangians it is sufficient to analyze the structure of the vacuum expectation values

$$\begin{aligned}\langle 0|\,TL_{\text{int}}(x_1)\ldots L_{\text{int}}(x_3)\,|0\rangle &= \prod_{1\leqq i<j\leqq 3} [e^{f^2 iD_F(x_i-x_j)}-1] \\ &\quad+\tfrac{1}{2}\sum_{\sigma\in\mathfrak{S}_3} [e^{f^2 iD_F(x_i-x_j)}-1]\,[e^{f^2 iD_F(x_j-x_k)}-1] \\ &= \prod_{1\leqq i<j\leqq 3} iE_F(x_i-x_j)+\tfrac{1}{2}\sum_{\sigma\in\mathfrak{S}_3}[iE_F(x_i-x_j)]\,[iE_F(x_j-x_k)]\,.\end{aligned} \tag{2}$$

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