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The Information on the Pion Electromagnetic Form Factor Inside Its Analyticity Region Provided by Bounds on Its Modulus on the Cut ($t \geqq 4m^2$)*

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Abstract. We concentrate on the mathematical aspects connected with the derivation of the model independent information one can get on the pion electromagnetic form factor $F(t)$ inside the analyticity region (the cut t-plane) from the knowledge of upper and lower bounds of its modulus on the cut $t \geqq 4m^2$ using analyticity, reality, and the normalization $F(0) = 1$. It turns out that (in a certain sense) this information depends only on the upper bound, whereas the lower one is irrelevant.

1. Introduction

We consider in this paper the mathematical steps involved in the derivation of the model independent information on the pion electromagnetic form factor $F(t)$, contained in the $e^+e^- \to \pi^+\pi^-$ scattering data.

The cross section of this process is given, up to a kinematic factor, by the square of the modulus of $F(t)$:

$$\sigma(t) = \mathrm{K}(t)\,|F(t)|^2, \qquad t \geqq 4m^2\,, \tag{1.1}$$

$$K(t) = \left(\frac{e^2}{4\pi}\right)^2 \frac{\pi}{3}(t-4m^2)^{\frac{3}{2}} t^{-\frac{5}{2}}, \qquad \frac{e^2}{4\pi} = \frac{1}{137} \tag{1.2}$$

where t has the meaning of the total energy squared in the c.m. system and m that of the pion mass.

We admit that $F(t)$ is a real analytic function in the complex t-plane cut along $t \geqq 4m^2$, normalized at $t = 0$ to $F(0) = 1$. The cross section (1.1) then provides us with data on the boundary values of the modulus of this function on the cut. These data take, due to the experimental errors, the form of upper and lower bounds,

$$|I(t)| \leqq |F(t)| \leqq |S(t)|, \qquad t \geqq 4m^2\,. \tag{1.3}$$

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