Commun. math. Phys. 26, 109—120 (1972)

Connection between the Spectrum Condition and the Lorentz Invariance of $P(\phi)_2$

R. F. STREATER

Bedford College, London, England

Received September 24, 1971; in revised form January 31, 1972

Abstract. We prove that, for the $P(\phi)_2$ quantum field theory, the Wightman functions are Lorentz invariant if the energy-momentum spectrum lies in the forward light-cone. The ingredients of the proof are the following facts, established by Glimm and Jaffe: the field satisfies local commutativity, and also the estimates

$$\phi_V(f, t) \leqq \text{const} \|f\|_1 (H_V + I)$$

$$\pi_V(g, t) \leqq \|g\|_2 (H_V + I)$$

where V is a space cut-off, uniformly in V.

1. Introduction

Glimm and Jaffe [1] have proved that for the $P(\phi)_{2,V}$ theory (the self-interacting boson quantum field theory in two-dimensional space-time, with a polynomial interaction and a periodic box cut-off, V) the canonical conjugate field $\pi_V(g, t) \equiv \int \pi_V(x, t)\, g(x)\, dx$ satisfies the estimate

$$\pm \pi_V(g, t) \leqq \|g\|_2 (H_V + I). \tag{1}$$

Here H_V is the Hamiltonian for the cut-off theory, and I is the identity operator. (There is a gap in the proof, in [1], of a similar estimate for $\nabla \phi_V$.) Furthermore [2] the field itself satisfies the estimate

$$\pm \phi_V(f, t) \leqq \text{const} \|f\|_1 (H_V + I) \tag{2}$$

where the constant is independent of V. These inequalities lead to bounds on vacuum expectation values of products of ϕ_V and π_V, showing that these expectation values are tempered distributions. Since the bounds are independent of V [2, 3] one obtains similar bounds for the smeared n-point Wightman distributions for the theory with no cut-offs. In particular, the Wightman function

$$W_n(z_1, \dots z_n) = (\Omega, \phi(z_1) \dots \phi(z_n)\, \Omega) \tag{3}$$