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Phase Transitions in Anisotropic Classical Heisenberg Ferromagnets*

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Abstract. The method of Peierls is used to prove the existence of a spontaneous magnetization for a spin system with nearest-neighbor interactions and Hamiltonian $H = -J \sum_{\langle ij \rangle} [S_i^z S_j^z + \alpha(S_i^x S_j^x + S_i^y S_i^y)]$, S_i a (classical) unit vector at the i'th site, with $|\alpha| < 0.0298$ and 0.0198 for a square lattice and simple cubic lattice, respectively.

I. Introduction

In the modern theory of phase transitions an argument due, in essence, to Peierls [1], and later made rigorous by Dobrushin [2] and Griffiths [3] has been very useful for proving the existence of phase transitions in several model systems [4, 5]. In particular Ginibre [6] (see also Robinson [13]) has considered the anisotropic Heisenberg model or quantum lattice gas with Hamiltonian

$$\begin{aligned} H &= U + K; \qquad U = -J \sum_{\langle ij \rangle} S_i^z S_j^z ; \\ K &= -J\alpha \sum_{\langle ij \rangle} (S_i^x S_j^x + S_i^y S_j^y), \end{aligned} \tag{1.1}$$

where S_i^x, etc. are the angular momentum operators (Pauli matrices) for a spin 1/2 atom located at a site i on a regular d-dimensional (hyper) cubic lattice ($d \geqq 2$), $\langle ij \rangle$ denotes a pair of nearest neighbor sites on the lattice (each pair counted in the sum only once), J is a positive constant and α a real number.

For this model (only one of several which he considered), Ginibre proved the existence of a phase transition – the two phases characterized by the thermal average $\langle S_i^z \rangle$ having positive and negative values, respectively, in the thermodynamic limit – for sufficiently low temperatures and provided $|\alpha|$ is sufficiently small. The actual estimates employed by Ginibre (which could, no doubt, be refined) require that $|\alpha|$ not exceed 10^{-6} for $d = 2$, with even smaller limits for larger d. On the other hand, Fisher [7] has given intuitive arguments which suggest that a phase transition probably occurs for any $|\alpha| < 1$, while Mermin and Wagner [8]

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